$$\frac{\csc t + \sin(-t)}{\cos(-t) - \sec t}$$

$$= \frac{\csc t - \sin t}{\cos t - \sec t}$$

$$= \frac{\frac{1}{\sin t} - \sin t}{\cos t - \frac{1}{\cos t}} \cdot \frac{\sin t \cos t}{\sin t \cos t}$$

$$= \frac{\cos t - \sin^2 t \cos t}{\sin t \cos^2 t - \sin t}$$

$$= \frac{\cos t\,(1 - \sin^2 t)}{\sin t\,(\cos^2 t - 1)}$$

$$= \frac{\cos t \cos^2 t}{\sin t\,(-\sin^2 t)}$$

$$= -\frac{\cos^3 t}{\sin^3 t} = -\cot^3 t$$

$$\frac{\cot^2\beta - \csc^2\beta}{1-\sin^2\beta}$$

$$= \frac{-1}{\cos^2\beta}$$

$$= -\sec^2\beta$$

Prove $\dfrac{1}{1-\tan x}+\dfrac{\tan x}{1-\cot x}=1+\tan x$.

$$= \frac{1}{1-\tan x} + \frac{\tan x}{1-\frac{1}{\tan x}} \cdot \frac{\tan x}{\tan x}$$

$$= \frac{1}{1-\tan x} + \frac{\tan^2 x}{\tan x - 1}$$

$$= \frac{1}{1-\tan x} - \frac{\tan^2 x}{1-\tan x}$$

$$= \frac{1-\tan^2 x}{1-\tan x}$$

$$= \frac{\cancel{(1-\tan x)}(1+\tan x)}{\cancel{1-\tan x}}$$

$$= 1+\tan x \quad \boxed{\text{QED}}$$

$$\sec^4\theta - \tan^2\theta = \sec^2\theta + \tan^4\theta$$

$$= (\tan^2\theta + 1)^2 - \tan^2\theta$$

$$= \tan^4\theta + 2\tan^2\theta + 1 - \tan^2\theta$$

$$= \tan^4\theta + \tan^2\theta + 1$$

$$= \tan^4\theta + \sec^2\theta \qquad \boxed{\text{QED}}$$

$$\frac{\cos y + \cot y}{1 + \sin y} = \cot y$$

$$= \frac{\cos y + \frac{\cos y}{\sin y}}{1 + \sin y} \cdot \frac{\sin y}{\sin y}$$

$$= \frac{\cos y \sin y + \cos y}{(1 + \sin y)\sin y}$$

$$= \frac{\cos y \cancel{(\sin y + 1)}}{\cancel{(1 + \sin y)}\sin y}$$

$$= \frac{\cos y}{\sin y}$$

$$= \cot y \quad \boxed{\text{QED}}$$